# Plasma Correlation Effects on Stark Broadening

H. Capes and D. Voslamber

Association EURATOM-CEA
Département de la Physique du Plasma
et de la Fusion Contrôlée
Centre d'Etudes Nucléaires, Fontenay-aux-Roses (France)

(Z. Naturforsch. **30 a**, 913–915 [1975]; received April 2, 1975)

The influence on spectral line profiles of a possible coupling between the low and high frequency parts of the plasma microfield, and of ternary plasma correlations is investigated. Both effects may have some importance in high density plasmas (e. g. arc experiments), but the first of them is shown to be generally predominant.

Theoretical treatments of plasma correlations in line broadening are generally based on binary models using the Debye-Hückel condition (assuming the Debye sphere to contain a large number of charged particles) and on the intuitive microfield model of Baranger and Mozer [1] (assuming the low-frequency (LF) part of the microfield not to be correlated with the high-frequency (HF) part).

Two questions arise when the number of particles in the Debye sphere is no longer very large compared to unity:

1) Do ternary and higher multiple correlations significantly affect the line profile?
2) Is the line profile influenced by a possible coupling between the LF and HF parts of the microfield? [2]

In order to investigate these problems, we have generalised our study on electron correlations [3] (hereafter referred to as I) to include the general case of an ion-electron plasma. Instead of using the model of Baranger and Mozer, the quantities $\Phi_s(1, 2, \ldots, s)$ and $f_s(1, 2, \ldots, s)$ of paper I have been extended in an obvious way to include the ion variables [$\Phi_{N_i s}(\mathbf{X}, 12\ldots s)$, $f_{N_i s}(\mathbf{X}, 12\ldots s)$, $\mathbf{X}$ standing for the $6N_i$ ion coordinates], and the corresponding ion potentials have been introduced into Eqs. (8) and (10) of I. The (generalised) BBGKY-like hierarchy (10) can then be truncated after the third ($s=2$) equation by excluding those four-particle correlations which concern the radiating atom and the electron perturbers, and in addition those three particle correlations which involve two simultaneous atom-electron interactions. Note that at this stage all interactions with and between ions are retained. After this closure procedure the third equation of the hierarchy may be used to calculate $\Phi_{N_i 2}(\mathbf{X}, 12)$. The result is

$$\Phi_{N_i 2}(\mathbf{X}, 12) = \Phi_{N_i 0}(\mathbf{X}) f_2(\mathbf{X}|12) + \{\Gamma(\mathbf{X},1) f_1(\mathbf{X}|2)\,[1+g(\mathbf{X}|12)] + (1 \leftrightarrow 2)\} + N_e f_1(\mathbf{X}|1) f_2(\mathbf{X}|2) \int \Gamma(\mathbf{X},3)\, g(\mathbf{X}|13)\, g(\mathbf{X}|23)\, d(3) + \Delta(\mathbf{X}, 12)\,. \quad (1)$$

Here $f_s(\mathbf{X}|12\ldots s)$ (a conditional probability function for $s$ electrons in the presence of a given configuration $\mathbf{X}$ of ions), $g(\mathbf{X}|12)$ and $(\mathbf{X}, 1)$ are defined by the following relations:

$$f_{N_i 0}(\mathbf{X}) f_s(\mathbf{X}|12\ldots s) \equiv f_{N_i s}(\mathbf{X}, 12\ldots s)\,,$$
$$f_2(\mathbf{X}|12) \equiv f_1(\mathbf{X}|1) f_1(\mathbf{X}|2)\,[1+g(\mathbf{X}|12)]\,,$$
$$\Gamma(\mathbf{X}, 1) \equiv \Phi_{N_i 1}(\mathbf{X}, 1) - \Phi_{N_i 0}(\mathbf{X}) f_1(\mathbf{X}|1)\,.$$

The quantity $\Delta(\mathbf{X}, 12)$, a lengthy expression which will not be given explicitly in this note, can be shown to provide a relative correction of order $\varepsilon = (4\pi\nu\lambda_D^3/3)^{-1}$ to the collission operator ($\lambda_D$ being the Debye length, $\nu$ the electron density). In the expression for $\Delta$ and in the subsequent relations, the following "screened" potentials are involved:

$$\hat{\overline{V}}_i(\mathbf{X}) = \sum_{j=1}^{N_i} \overline{V}_i(j) + N_e \int \overline{V}_e(1)\, f_1(\mathbf{X}|1)\, d(1)\,, \quad (2\,a)$$

$$\hat{\overline{V}}_e(\mathbf{X}|1) = \overline{V}_e(1) + N_e \int \overline{V}_e(2)\, f_1(\mathbf{X}|2)\, g(\mathbf{X}|12)\, d(2)\,, \quad (2\,b)$$

$$\hat{W}_{ei}(1, \mathbf{X}) = \sum_{j=1}^{N_i} W_{ei}(1, j) + N_e \int W_{ee}(12)\, f_1(\mathbf{X}|2)\, d(2)\,, \quad (2\,c)$$

$$\hat{W}_{ee}(\mathbf{X}|12) = W_{ee}(12) + N_e \int W_{ee}(13)\, f_1(\mathbf{X}|3)\, g(\mathbf{X}|23)\, d(3)\,. \quad (2\,d)$$

Here $\overline{V}_i$ and $\overline{V}_e$ denote respectively the interaction Hamiltonians of the "doubled atom" (see I) with an ion and an electron; $W_{ei}$ and $W_{ee}$ are the electron-ion and electron-electron Coulomb potentials.

Reprint requests to Dr. H. Capes, Association Euratom-CEA, Département de la Physique du Plasma et de la Fusion Controlée, Centre d'Etudes Nucléaires, *F-92260 Fontenay-aux-Roses* (France).

We now may insert expression (1) into the second equation of the hierarchy, which then serves to determine $\Phi_{N_i 1}(\mathbf{X}, 1)$ and, with the help of the first equation, the quantity $\Phi_{N_i 0}(\mathbf{X})$. The Laplace transform of the latter quantity with respect to time, $\tilde{\Phi}_{N_i 0}(\mathbf{X}, p)$, can be easily calculated in this way on making the quasi-static approximation for the ion perturbers. It is obtained from the following result:

$$[p + (i/\hbar)(\overline{H}_0 + \hat{\overline{V}}_{\mathrm{i}}(\mathbf{X})) + K_{N_i}(\mathbf{X}, p)]\, \tilde{\Phi}_{N_i 0}(\mathbf{X}, p) = f_{N_i 0}(\mathbf{X}) \tag{3}$$

where

$$K_{N_i}(\mathbf{X}, p) = -\frac{N_{\mathrm{e}}}{\hbar^2} \int \mathrm{d}(1)\,\mathrm{d}(2)\, \overline{V}_{\mathrm{e}}(1)\, G'(\mathbf{X} | \mathbf{r}_1 \mathbf{r}_2, p) \int_0^\infty \mathrm{d}t\, e^{-pt} Z_2(\mathbf{X}, t)\, Q(\mathbf{X}, 2, t)\, f_1(\mathbf{X}|2)\, \hat{\overline{V}}_{\mathrm{e}}(\mathbf{X}|2)\,.$$

Here, the propagator $Z_2$ obeys the equation

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v}_2 \cdot \frac{\partial}{\partial \mathbf{r}_2} - \frac{1}{m_{\mathrm{e}}} \frac{\partial \hat{W}_{\mathrm{ei}}(2, \mathbf{X})}{\partial \mathbf{r}_2} \cdot \frac{\partial}{\partial \boldsymbol{v}_2} + S(2)\right] Z_2(\mathbf{X}, t) = 0$$

with $Z_2(\mathbf{X}, 0) = 1$, where $S(2)$ is a diffusion type operator depending on $\Delta(\mathbf{X}, 12)$, and the time evolution operator $Q$ satisfies the equation

$$\left[-i\hbar \frac{\partial}{\partial t} + \overline{H}_0 + \hat{\overline{V}}_{\mathrm{i}}(\mathbf{X}) + Z_1^{-1} \hat{\overline{V}}_{\mathrm{e}}(1) Z_1\right] Q(\mathbf{X}, 1, t) = 0\,.$$

Further, the Fourier transform of the Green's function $G'$ is given by the relation (25) of I, where the Fourier transform of $D$ has to be replaced by the expression

$$D'(\mathbf{X} | \mathbf{r}_1 \mathbf{r}_2, p) = \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{N_{\mathrm{e}}}{m_{\mathrm{e}}} \int_0^\infty \mathrm{d}t\, e^{-pt} \int Z_1(\mathbf{X}, t)\, Q(\mathbf{X}, 1, t) \cdot [1 + g(\mathbf{X}|12)]\, \frac{\partial W_{\mathrm{ee}}(12)}{\partial \mathbf{r}_1} \cdot \frac{\partial f_1(\mathbf{X}|1)}{\partial \boldsymbol{v}_1}\, \mathrm{d}^3 v_1\,. \tag{4}$$

In order to perform the average over ion configurations, we introduce a projection operator acting on functions of $\mathbf{X}$,

$$P(\ldots) = [f_{N_i 0}(\mathbf{X}) / w(\mathbf{E})] \int (\ldots)\, \delta[\mathbf{E} - \mathbf{E}_i(\mathbf{X})]\, \mathrm{d}\mathbf{X}$$

where $\mathrm{w}(\mathbf{E}) = \int f_{N_i 0}(\mathbf{X})\, \delta(\mathbf{E} - \mathbf{E}_{\mathrm{i}}(\mathbf{X}))\, \mathrm{d}\mathbf{X}$ is the low frequency microfield distribution[4, 5], $\mathbf{E}_i(\mathbf{X})$ denoting the screened ion microfield which appears in $\hat{\overline{V}}_{\mathrm{i}}(\mathbf{X})$ after making the dipole approximation.

By applying $P$ and $(1-P)$ to Eq. (3) we obtain two coupled equations for $P\tilde{\Phi}_{N_i 0}$ and $(1-P)\tilde{\Phi}_{N_i 0}$ leading to the solution

$$\tilde{\Phi}_{00}(p) = \int w(\mathbf{E}) \left[p + \frac{i}{\hbar}(\overline{H}_0 + \overline{\mathbf{D}} \cdot \mathbf{E}) + K(\mathbf{E}, p)\right]^{-1} \mathrm{d}^3 E$$

with

$$K(\mathbf{E}, p) = \frac{-1}{w(\mathbf{E})} \int \delta[\mathbf{E} - \mathbf{E}_i(\mathbf{X})]\, K_{N_i} \left[p + \frac{i}{\hbar}(\overline{H}_0 + \overline{\mathbf{D}} \cdot \mathbf{E}) + (1-P) K_{N_i}\right]^{-1} f_{N_i 0}(\mathbf{X})\, \mathrm{d}\mathbf{X} \cdot \left[p + \frac{i}{\hbar}(\overline{H}_0 + \overline{\mathbf{D}} \cdot \mathbf{E})\right]. \tag{5}$$

The presence of the term $(1-P)K_{N_i}$ in this expression is due to multiple plasma interactions. Its effect can be evaluated by means of a series expansion and gives rise to relative contributions to $K(\mathbf{E}, p)$ at most of the order of $\varepsilon$. Apart from these contributions, Eq. (5) takes the form of an average of $K_{N_i}$ which, leaving aside again terms of order $\varepsilon$, can be simplified further by omitting $\Delta$ in Eq. (1), by suppressing the dependence of $\mathbf{X}$ in Eqs. (2 b) and (4), and by putting $g(\mathbf{X}|12) = 0$ in Eq. (4). The following result is then obtained

$$K(\mathbf{E}, p) = -\frac{N_{\mathrm{e}}}{\hbar^2} \int V_{\mathrm{s}}(\mathbf{r}_1, p)\, \mathrm{d}(1) \int_0^\infty e^{-pt} \langle Z_1(\mathbf{X}, t)\, Q(\mathbf{X}, 1, t)\, f_1(\mathbf{X}|1) \rangle_{\mathbf{E}}\, V_{\mathrm{s}}(\mathbf{r}_1, 0)\, \mathrm{d}t \tag{6}$$

where $V_{\mathrm{s}}(r, p)$ is defined by Eq. (33) of I and $\langle \ldots \rangle_{\mathbf{E}}$ stands for an average with the distribution function $f_{N_i 0}(\mathbf{X})\, \delta(\mathbf{E} - \mathbf{E}_i(\mathbf{X})) / w(\mathbf{E})$. Note that the propagator $Z_1$ acts on all quantities to its right, including $V_{\mathrm{s}}(\mathbf{r}_1, 0)$.

Our result (6) differs in two respects from the corresponding expression (34) of I.

Firstly, the electron-atom interaction potential involved in $Q$ is given by the statically shielded expression $V_s(\boldsymbol{r}_1, 0)$. However, the relative contribution to the collision operator of the shielding in $Q$ is at most of order $\varepsilon$, and it would not be consistent to retain this contribution in a treatment where other terms of order $\varepsilon$ are neglected. Note that even if the shielding in $Q$ were significant, our result would not confirm a conjecture in ref. [6] concerning the use of Rostoker's quasi-particles, because $V_s(\boldsymbol{r}, 0)$ is not shielded dynamically.

The second difference between Eq. (6) and Eq. (34) of I lies in the appearance of the average $\langle Z_1 Q f_1 \rangle_{\boldsymbol{E}}$ which shows an influence of the LF microfield component on the electron distribution function and on the trajectories of the electron perturbers. This effect can be summarized in terms of an "inhomogeneous" electric force field associated with the LF field $\boldsymbol{E}$ at the atom. This force acts on the electron perturbers and is given by

$$\boldsymbol{F}_{\boldsymbol{E}}(\boldsymbol{r}_1) = - \frac{\partial}{\partial \boldsymbol{r}_1} \langle \hat{W}_{ei}(1, \boldsymbol{X}) \rangle_{\boldsymbol{E}} .$$

An evaluation of this effect for the central region of hydrogen lines yields a relative contribution to the collision operator of the order of $\varepsilon^{2/3}/\ln(\lambda_D/b_w)$ ($b_w$ being the Weisskopf radius), with a numerical factor approximately equal to plus one. Since $\varepsilon$ is small compared to unity in the validity domain of the theory, it is consistent to retain this contribution while neglecting the corrections of order $\varepsilon$ mentioned above. This contribution may have some importance for cases where $\varepsilon$ is small, but not very small compared to one (e. g. in arc experiments [7]) and might, at least partially, explain some of the remaining discrepancies between theoretical and experimental line shapes.

[1] M. Baranger and B. Mozer, Phys. Rev. **115**, 521 [1959].
[2] G. Ecker and W. Kröll, in Proceedings of the IXth Int. Conf. Phen. Ion. Gases, Bucharest 1969 (unpublished).
[3] H. Capes and D. Voslamber, Phys. Rev. **A 5**, 2528 [1972].
[4] H. Pfennig and E. Trefftz, Z. Naturforsch. **21 a**, 697 [1966].
[5] C. F. Hooper, Jr., Phys. Rev. **165**, 215 [1968].
[6] E. W. Smith, J Cooper, and C. R. Vidal, Phys. Rev. **185**, 140 [1969].
[7] W. L. Wiese, D. E. Kelleher, and D. R. Paquette, Phys. Rev. **A 6**, 1132 [1972].